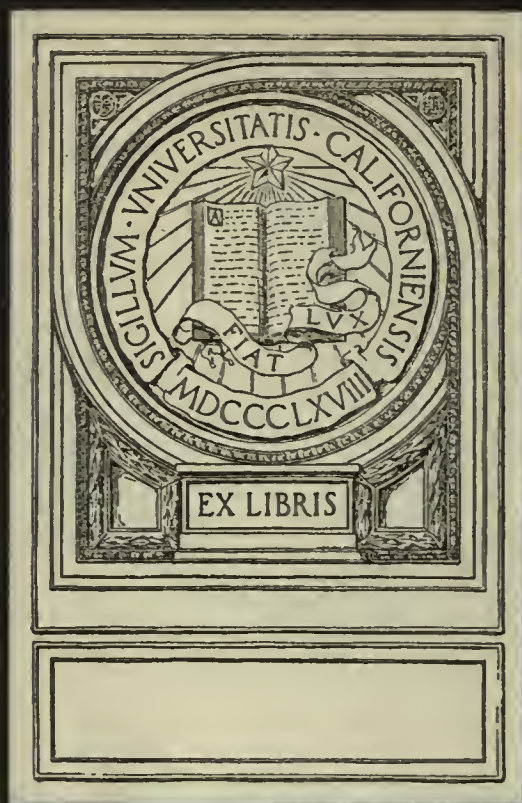


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DIAGNOSTIC AND REMEDIAL TREATMENT FOR ERRORS IN ARITHMETICAL REASONING

By

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DIAGNOSTIC AND REMEDIAL TREATMENT FOR ERRORS IN ARITHMETICAL REASONING

INTRODUCTION

A voluminous literature has appeared in recent years based upon the correct responses which children make. The last decade will be remembered in educational history as the time when large numbers of standard tests first began to be published, and when the rank and file of the teaching profession began to get acquainted with the application of elementary statistical procedure to the field of education. Splendid as this movement has been it is by no means complete. The purpose of this and other circulars which are to follow is to describe, in some measure, a further line of attack upon our educational problems based on the study of wrong responses.

There is good argument to support the thesis that a study based upon wrong responses is likely to be even more stimulating and helpful than a procedure based upon the results of the child's successful efforts. In the business world much study is given to the points at which the output of the factory fails to function as it should. The manufacturers of automobiles, for example, have very definite information concerning the weaknesses of their cars. From a study of these weaknesses it is possible to improve the output. In general a contact with failures and breakdowns stimulates thinking. Improvements can come only when there is a somewhat definite knowledge of failures.

Progress in the study of errors has long been retarded by the idea that there is the utmost diversity in the mistake of children. The actual evidence is far to the contrary. The same wrong answers occur in all types of schools and over wide areas to such an extent that it is possible to forecast how many pupils in any individual school will make certain mistakes. Errors are like diseases in some respects. There are a great many different diseases, but only a few of them are wide-spread and frequent in occurrence.

In like manner the evidence so far available shows that four or five different things are responsible for at least three fourths of the trouble which children have. To identify these widespread errors, determine their causes, and plan their cure is clearly an important task in educational research. Such data will lead to more intelligent supervision, more scientific methods of teaching, better organization of curriculum material, and more efficient teacher training.

With these facts in mind it is rather surprising to find how little is available in print concerning the nature and cause of failure. This circular is one of a series begun last year and designed to investigate these questions as they apply to the several school subjects.

THE IMPROVEMENT OF ABILITY IN ARITHMETICAL REASONING

In discussing a remedial program on Arithmetical Reasoning the following items must be known and agreed upon:-

1. A list of typical problems which have been widely used and on which lists of wrong answers are available.
2. An identification and classification of the most serious disabilities.
3. A list of the wrong answers which are symptomatic of each disability in each type of problems.
4. The remedy to apply after the specific disabilities are known and identified.

Each of these will be considered in order.

1. A list of the Problems which were used.

In this study the problems in the Buckingham Problem Test, Form 1 were used.

- (1) We learn 2 words a day in our class. How many do we learn in 8 days?
- (2) 23 children belong to our class but only 19 are present. How many children are absent?
- (3) James has 28 marbles. He gives half of them to Charles. How many has he left?
- (4) If you can get 3 ginger-bread dogs for 5 cents, how many can you get for ten cents?
- (5) A boy owned 3 kites, each of them having 150 feet of string. How many feet of string had he?
- (6) A baseball team took 12 players on a trip. The trip cost the team \$36.00. How many was that for each player?
- (7) An automobile was run 30 miles every day for a week. How many miles did it go?
- (8) Henry gathered 5 quarts of nuts. He sold them at 8 cents a quart and spent the money for oranges at 4 cents apiece. How many did he buy?
- (9) If an electric car runs 9 miles an hour, how many hours will it take to travel from one city to another, 117 miles away?
- (10) Ned sold his rabbit for 30 cents. This was $\frac{3}{5}$ of what he paid. What did he pay for the rabbit?
- (11) If a girl had two one-dollar bills, three five-cent pieces, two dimes, and three quarters, how much money did she have?
- (12) How many pencils can you buy for 50 cents at the rate of 2 for 5 cents?
- (13) A boy had 210 marbles. He lost $\frac{1}{3}$ of them. How many were left?
- (14) Two tubs of maple sugar weighed 42 lbs. One weighed $18\frac{1}{4}$ lbs. How many pounds did the other weigh?

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- (15) A store takes in the following sums: \$1250, \$300, \$175, \$16.25, \$120.50, \$32.75, \$68.50. It pays out \$600, \$360, \$166.67, \$33.33, \$240. How much remains after payments are made?
- (16) A man bought a house for \$7250. After spending \$321.50 for repairs, he sold it for \$9125. How much did he gain?
- (17) How many weeks will it take Joseph to save 21 dollars for a bicycle if he saves 1 1/2 dollars a week?
- (18) George walks at the rate of 2 1/3 miles per hour. Henry starts with him and walks in the same direction at the rate of 3 miles an hour. How many miles apart will they be in 3 hours?
- (19) If 0.78 of the weight of potatoes is water, how many pounds of water are there in a bushel of potatoes? if a bushel of potatoes weighs 60 pounds?
- (20) Two boys agreed to cut a lawn for 90 cents. The first boy worked one hour and the second boy worked 4 hours. How much money should each receive?
- (21) Find the total cost of the following purchases: 8 1/2 yds flannel at 96 cents, 4 3/4 yds braid at 16 cents, 12 yds. embroidery at 22 1/2 cents, 10 yds. lace at 27 1/2 cents.
- (22) The distance from New York to Chicago is 993 miles. The running time, by the New York Central is 12 hours. Because of floods, an engine goes only 296 miles the first 7 hours. After that how many miles per hour must it go to reach Chicago on time?
- (23) A, B, and C owned all the stock in a certain store worth \$4,800. A owned 16 shares, B 3 shares, and C 5 shares. What was the money value of each person's share of the stock?
- (24) A real estate dealer made a profit of \$5,000 on a plot of ground. The rate of profit was 20%. Find the cost of the plot.
- (25) It is estimated that if the house fly were kept from food, deaths in cities in summer would be 1/3 less. If the deaths in the summer months are at the rate of 12 per 1,000 people, how many lives might thus be saved in one summer in a city of 1,000,000 inhabitants?
- (26) I bought a cask of molasses containing 84 gallons for \$28. Nine gallons having leaked out, at what price per gallon must I sell the remainder to gain \$4.25?
- (27) A contractor completed 2/5 of a job in 12 1/2 days. How many days longer should it take to finish the job?
- (28) In the manufacturing of jute twine, the raw jute costs 8 3/5 cents a pound, the spinning cost 7/8 of a cent a pound, and the twisting and finishing cost 15/16 of a cent a pound. Find the cost of 1,500 pounds of jute twine.
- (29) On a map, a line 2 3/4 inches long represents a distance of 132 miles. How many miles are represented by a line 8 1/2 inches long?

- (30) Mr. Wood had 250 baseballs. He sold 20% of them the first day and 40% of the remainder the second day. What per cent of his original stock was left?

2. Identification and Classification of the most serious disabilities

The following classification and nomenclature is tentative only. It is based on a study of about 30,000 errors made by 6000 children in eighteen counties and one large city. The percents show how frequently each type of error occurred. The classification is as follows:-

	<u>Approximate Frequency</u>
(1) Total failure to comprehend the problem.....	30%
(2) Procedure partly correct but with the omission of one or two essential elements	20%
(This type will be designated as partial response hereafter)	
(3) Ignorance of fundamental quantitative relations	10%
Total	60%
(4) Errors in fundamentals	20%
(5) Miscellaneous errors	2%
(6) Errors whose cause could not be discovered ,.....	18%
Total	100%

3. Symptoms of the Several Disabilities.

When tests have been given and the results are poor, the supervisors and teachers will want to know how to tell which disability is causing the trouble.

In the following summary a list of the more serious disabilities is given together with the wrong answers which are the symptoms of each. When a pupil gets a wrong answer to a problem in this test look for the wrong answer in the list which follows. If it appears here the name of the disability which it represents will also be given.

The Typical Wrong Answers And The Disabilities of Which They Are The Symptoms.

<u>Problem</u>	<u>Wrong Answers</u>	<u>Disability</u>
(1)	10, 4, 6,	Total failure to comprehend
	18, 14, 15	Errors in the fundamentals
1	8, 2	Partial response

<u>Problem</u>	<u>Wrong Answers</u>	<u>Disability</u>
(2)	42, 437, 1 4/19	Total failure to comprehend
	3, 14, 16, 5, 6	Errors in fundamentals
(3)	13, 18, 24, 12, 15, 16	Errors in fundamentals
(4)	18, 15, 50	Total failure to comprehend
	2, 30,	Partial response
(5)	153, 50	Total failure to comprehend
	453, 350, 550, 553, 250	Errors in fundamentals
(6)	\$432, \$24, \$48	Total failure to comprehend
	\$2, \$4, \$5, \$6	Errors in fundamentals
(7)	180, 150 120, 160	Ignorance of quantitative relations
	100	Total failure to comprehend
	217	Errors in fundamentals
	30, 7	Partial response
(8)	17, 32, 20	Total failure to comprehend
	2, 40, 5, 8, 4	Partial response
	11,	Errors in fundamentals
(9)	1053, 126, 108	Total failure to comprehend
	10, 11, 14, 19	Errors in fundamentals
(10)	38, 30 3/5, 15, 18, 90. 6	Total failure to comprehend
	20, \$1.50, 30, 10	Partial response
(11)	\$2.00, \$1.10, \$2.85	Partial response
	\$3.00, \$ 3.20, \$3.05, \$2.10	
	\$1.12	Errors in fundamentals
(12)	40, 125, 10, 25, 250	Total failure to comprehend
	100, 30	Partial response
(13)	209, 2/3, 630, 105, 207	Total failure to comprehend
	70	Partial response
	130, 137	Errors in fundamentals

<u>Problem</u>	<u>Wrong Answers</u>	<u>Disability</u>
(14)	766 $1/2$, 60 $1/4$, 2 $22/73$, 21 24 $1/4$, 24 $3/4$, 34 $3/4$, 34, 23, $1/2$, 34 $1/4$, 24 $1/2$ 42, 18 $1/4$	Total failure to comprehend Errors in fundamentals Partial response
(15)	\$1563, \$573, \$562.80, \$1463, \$463, \$562, \$563.25, \$573, \$553, \$43.25 \$1963	Errors in fundamentals Partial response
(16)	\$1875 \$16696.50 \$1453.50, \$1653750, \$1554.50, \$463, \$553.50, \$302.75 2196.50	Partial response Total failure to comprehend Errors in fundamentals Ignorance of quantitative relations
(17)	31 $1/2$, 22 $1/2$, 19 $1/2$, 20, 63 2, 10 $1/2$, 10, 21, 7, 42. 13, 15	Total failure to comprehend Partial response Errors in fundamentals
(18)	1, 6, 8 $1/3$, 1 $1/3$, 3 $1/3$, 2 $4/7$, 21, $2/3$, 5 $1/3$ 7, 9, 2 $1/3$, 3	Total failure to comprehend Partial response
(19)	.18, 1.3, 74, 4680, 138 46.7, 46.36, 47.58, 44.8, 43.8	Total failure to comprehend Errors in fundamentals.
(20)	\$4.50; .90 and \$3.60; .18; .20; .22 $1/2$ and .67 $1/2$; .90; \$3.60 .45	Total failure to comprehend Partial response
(21)	\$14.21, \$14.28, \$14.52, \$14.39, \$16.11 \$13.61, \$1.62	Errors in fundamentals Partial response
(22)	702, 52 $12/19$ 42. $2/7$, 3 $55/148$ 63 $9/11$	Partial response Total failure to comprehend Errors in fundamentals

<u>Problem</u>	<u>Wrong Answer</u>	<u>Disability</u>
(23) \$300, \$1600 and \$960		Total failure to comprehend
\$200; \$768, \$144 and \$240		Partial response
\$3200, \$600 and \$9000		Errors in fundamentals
(24) \$1000, \$6000, \$10000, \$100000, \$5020, \$4000		Total failure to comprehend
\$350000, \$25000, \$250		Errors in fundamentals
(25) 12000, 8333 $\frac{1}{3}$, 1000, 8,996000, 1200		Partial response
333 $\frac{1}{3}$, 666 $\frac{2}{3}$, 3333 $\frac{1}{3}$, 66666 $\frac{2}{3}$		Failure to comprehend
40000, 400, 4000000		Errors in fundamentals
(26) 39, \$32.25 .06		Partial response
(27) 5, 31 $\frac{1}{4}$, 50, 7 $\frac{1}{2}$		Total failure to comprehend
12 $\frac{1}{2}$, 37 $\frac{1}{2}$, 25		Partial response
(28) \$150, \$ 15000, \$156.18 $\frac{3}{4}$, \$155.93; \$156.11 $\frac{1}{4}$, \$156.10 $\frac{1}{4}$, \$155.96, \$156		Errors in fundamentals
(29) 353, 396		Total failure to comprehend
1122, 132, 528		Partial response
(30) 20 , 190, 100		Total failure to comprehend
40, 50, 60, 120		Partial response

General Symptoms

The disability which I have called total failure to comprehend, is indicated in general by the fact that the child merely juggles with the figures. If, in any problem, his answer is gotten by some aimless combination of some or all the numbers in the problem, it is safe to assume that he is suffering from total inability to comprehend.

Partial response means that the child overlooks or fails to respond to parts of the problem. For example, in the Problem - "A boy had 219 marbles and lost $\frac{1}{3}$ of them. How many had he left?" An answer of 70 shows that the pupil overlooked or failed to respond properly to the words had he left.

Where the answer is wrong in only a single digit, or where the decimal point is placed wrongly or omitted, it is safe to assume that the child has failed because of a mistake in the fundamentals. In a large number of cases the latter type of error combines with one of the two preceding types so that the child gets an answer only similar to those listed under the first two types. In this report these errors are reported under types 1 or 2 only and not under type 3

4. Remedial Treatment.

Some of the principles of Successful Remedial Treatment have already been suggested. It is essential that we have as accurate an idea as possible concerning the nature of the disability, and its extent. It is also very desirable to make the pupil conscious of the nature of his difficulty, and of what he must do to remove it. Much emphasis has been placed upon this last point by the school authorities at Winnetka, Illinois and Los Angeles, California. In these cities it has been found very much worth while to make the pupil thoroughly conscious of the "goal" which he must reach, and of the process by which he can reach it.

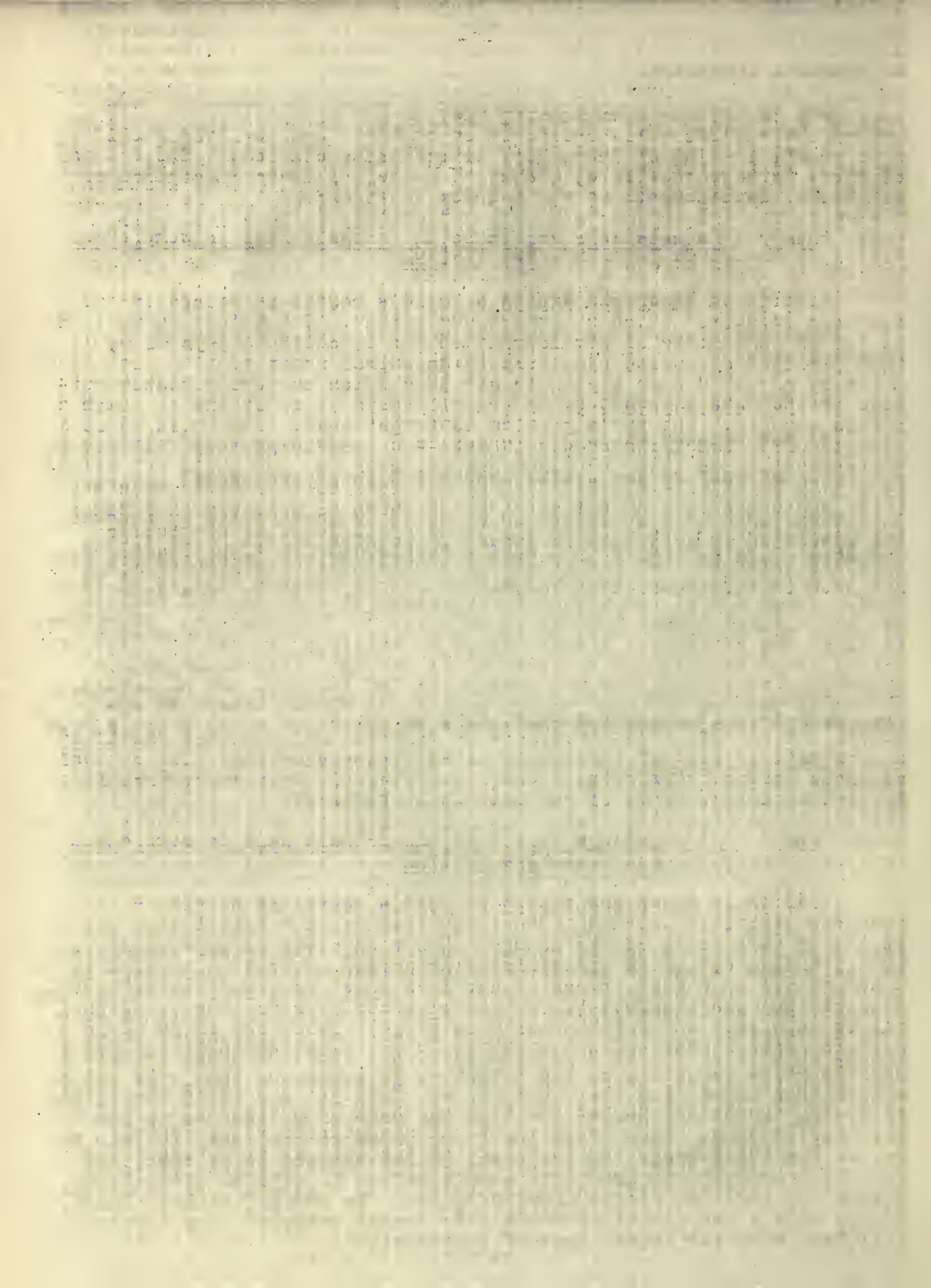
These three questions should be used to test our efforts at remedial work:-

- (1) What is the nature of the difficulty?
- (2) How serious is it?
- (3) How well do the pupils understand their own needs?

With these test questions in mind, it is worth while to discuss the situation in arithmetical reasoning in some detail. The frequencies given on page 4 show that about 60% of the wrong answers are due to a failure right in the beginning to understand what is to be done. About 20% of the children understand what to do, but fail to get the correct answer because of an error in fundamentals. Therefore, if these two sources of trouble could be entirely removed our list of errors would be only 1/5 as large as it now is. This would mean a tremendous saving of time and energy for the teacher, and an untold amount of gain on the part of the pupils. It should always be remembered in this connection that the discouragement which results from continual failure without knowing why, tends to impair in a serious manner the child's self-confidence in general, and may even result in a permanent feeling of incapacity (inferiority complex). The removal of these errors, therefore, becomes a matter of major importance in the future developments of our methods of teaching.

Remedial Suggestions For Those Who Totally Fail to Comprehend The Problem

Little is known concerning a child's mental reactions in the face of a situation which he cannot understand. The safest assumption at the present stage of development in our educational psychology is that a child acts quite similarly to an animal under similar situations. A hungry rat in a cage trying to get food which is some distance outside of the cage responds by making all sorts of reactions of which a rat is capable. The rat's problem is to get food. The child feels a need to write something on his tablet -- do something that will get a number that looks like the right answer. Both rat and child are blindly trying to satisfy needs. Neither has much of an idea as to how this may be done. Each tries the thing which is most easy for him to do, -- the thing which, in the past, he has learned to do best, and each will continue to make this same reaction to similar situations unless the reaction proves unsatisfactory. In short, whenever human beings meet a new situation which they cannot analyze, they invariably fall back upon the animal type of behavior in



such circumstances. In educational psychology this method of meeting new situations is called trial and error or fumbling and success. Our problem therefore is to substitute a higher type of learning for a lower one. This is the essence of teaching pupils to think and reason. We must meet this problem in some manner.

Prerequisites For Thinking

Specific directions on how to teach children to think are not yet available, but a few points are clear enough to be stated somewhat definitely. They are as follows:-

1. A supply of free ideas or concepts must be provided.
2. The ability to compare, combine, and judge the relative value of these ideas for the particular situations is necessary.
3. All of the relevant ideas must function, but none that are irrelevant can enter into the thinking process.
4. Relevant ideas must be encouraged and rewarded; those that are irrelevant must be discouraged and penalized.
5. Thinking is impossible until adequate experience has been provided in trial and error and imitation methods. The amount which may be called adequate varies according to the intelligence and past experience of the child.

The foregoing may be regarded as well established principles in teaching of thinking. The present problem of the field worker in education is to develop a technique which will assist in the realization of the needs as they are now known.

Some things to do for children who show a total failure to comprehend the problem are as follows:-

To increase the supply of free ideas or concepts.

Find whether or not the child is reading the problem correctly. This can be done in a very satisfactory manner by using the problems as silent reading material before any attempt is made at solution.

Samples of silent reading exercises which are prerequisite to the solution of Problems in Arithmetic.

To the pupils: Can you understand your arithmetic problems when you read them? If so, you can do these exercise correctly. After each problem is a list of things which might be done. You are to show that you understand by drawing a line under the word or words which tell the right thing to do.

- (1) Mary wishes to save 35 cents; if she has already saved 24 cents, how much more must she save?

Add

Subtract

Multiply

Divide

- (2) How many tablets can I buy for 40 cents, if each tablet costs 8 cents?

Add Subtract Multiply Divide

- (3) Fred's mother has 3 rows of fruit jars with 9 jars in each row. How many fruit jars has she?

Add Subtract Multiply Divide

- (4) Frank had 2 apples and his mother gave him 3 more. How many apples did he then have?

Add Subtract Multiply Divide

Fill in the blanks and underline the words which tell the right things to do.

- (5) How much cheaper is it to rent a room for a year at \$12.50 a month than it is to rent another room for the same length of time at \$3.50 a week?

Divide _____ by _____

Divide _____ by _____

Multiply _____ by _____

Multiply _____ by _____

Then Add Subtract Multiply Divide

- (6) A farmer sold 3375 bushels of potatoes at \$0.90 a bushel, and wishes to buy land at \$75 an acre. How many acres can he buy?

Divide _____ by _____

Multiply _____ by _____

Multiply _____ by _____

Then Add Subtract Multiply Divide

- (7) If a man receives \$7.20 for 8 hrs. work, how much should he get for 6 1/2 hrs. work?

Add _____ to _____ Multiply _____ by _____

Subtract _____ from _____ Divide _____ by _____

Then Add Subtract Multiply Divide

Exercises like the above will insure against careless reading. If the child fails to respond correctly after a short practice upon exercises of this sort, it may be taken as a sure sign that another type of prerequisite training is necessary. Such children have not had sufficient experience with concrete things. Free ideas are lacking and must be obtained, if at all, through the actual handling of the objects. They will need repeated practice of this sort which deals at first with one step problems only.

In problem 1 above it will be necessary to provide toy or real money. In problem 2 actual apples will be needed and so on. In general these children will be lacking in intelligence also. Much time and patience will probably be needed. When success has been attained with the simple problems more complicated ones should be used.

Some things to do for children who give only partial responses to the problem.

Most of the trouble here is due to a failure to read the problem completely. Exercises like the following will prove helpful.

To the pupil: Some boys and girls fail to get their problems because they do not see everything. How sharp are your eyes? You will get caught in the following exercises if you are not careful.

- (1) A boy had 240 marbles and lost $1/4$ of them. How many had he left?

Are you asked to find how many marbles the boy lost?

- (2) John sold his rabbit for 20 cents which was $4/5$ of what the rabbit cost him. How much did he pay for the rabbit?

Underline the words which make the following statement true.

John sold his rabbit for (more than, less than) it cost him

Are you asked to find out how much money John lost on his rabbit?

Underline the word which tells the right things to do.

Add

Subtract

Multiply

Divide

Some things to do for children who fail to understand quantitative relations.

Make a list of the important quantitative relations which occur in the problems which you assign. The following list is suggestive:

- (1) The relation between cost, expenditure, income, and selling price.

A certain house rents for \$960 a year, the taxes and upkeep cost \$300 a year. How much can I afford to pay for this house in order to clear 6% a year on my investment? Suppose I have bought the house at this price and am offered \$12,000 for it. Should I sell? Why?

- (2) The relation between rate, time, and distance.

The distance from New York to Chicago is 980 miles. How long will it take a train to run this distance at the rate of 49 miles per hour?

- (3) The relation between length, breadth, and thickness.

The first thing I noticed when I stepped out of the car was the cold. It was a sharp contrast to the warm blanket I had been sitting under. I looked up at the sky, which was a pale, hazy blue. The air was crisp and clean, a welcome change from the stuffy atmosphere of the car. I took a deep breath, feeling the cool air fill my lungs. The sun was just beginning to rise, casting a soft, golden glow over the landscape. The trees were still, their branches bare and reaching out towards the sky. The ground was covered in a thin layer of snow, which glistened in the early morning light. I walked slowly, my boots crunching against the snow. The silence was peaceful, a rare moment of quiet in a busy world. I felt a sense of calm, a sense of being in the right place at the right time. The world was beautiful, and I was grateful to be here.

I had heard that the weather was perfect, but I didn't realize how much I needed it. The cold was just what I needed to wake up. The sun was just what I needed to see the world in a new light. The snow was just what I needed to feel like a child again. I was in luck. I was in luck. I was in luck.

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I need a bin that will contain 240 cubic feet. If I make it 8 feet long and 6 feet wide, how deep must it be?

(4) Relations involving proportion.

It takes 9 men 5 days to build a piece of road. How many men would it take to build this road in 3 days?

Problems of this sort are of great practical value and are a serious source of error in both Arithmetic and Algebra. The best method of teaching them is not known. It is the belief of the writer that much help can be gotten for this type of difficulty by teaching the equation in arithmetic. Simple equations are not beyond the comprehension of most elementary school children. Some children will not understand them, but these children will not understand any method. Even with them the equation form is desirable because it can be taught in a mechanical form, the meaning of which the child will appreciate later. The following equations relate to the four problems given above. Only the first letters of the words are used.

1. I (income) - E (expenditure) = C (amount cleared)

C (cost) $\mp$ G (gain) = S (selling price)

2. D (distance) = R (rate) $\times$ T (time)

3. C (contents) = L (length) $\times$ B (breadth) $\times$ D (depth)

4. $9 \times 5 = ? \times 3$

In case G is wanted as in Number 1, start with $C \mp G = S$. Then teach that C which is not wanted on the same side with G can be put over with S by changing its sign as in $G = S - C$.

In case D is wanted as in Number 3, start with $C = L \times B \times D$. Then teach that D can be made to stand alone by putting L and B in the other side and changing their signs as in $C \div L \div B = D$.

Some things to do for children who make errors in the fundamental processes.

This topic is discussed in a circular of this Department entitled; "A Study Of Errors In Arithmetic", and will be treated further in a series of circulars on "Inventory", and "Remedial Exercises In Arithmetic", which are now in preparation.

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